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Reliability Analysis of Two-Unit Warm Standby System Subject to Hardware and Human Error Failures

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Abstract: In this study we investigated the probabilistic analysis of a two-unit warm standby system with subject to hardware and human error failures. The aim of the present paper is to study the probabilistic analysis of the system model. All repair time distributions are taken general where as all failure time distributions are taken as exponential type. The first repairman, usually called regular repairman, always remained with the repair facility, with the known fact that he might not be able to do some complex repairs within some tolerable (patience) time. An expert repairman was called for the system if and only if the regular repairman is unable to do the job, within some fixed time or on system failure, whichever occurs first. By using regenerative point technique, we analyses this system Steady state transition probabilities, Mean sojourn time, Reliability and mean time to system failure, Point wise and steady state availability of the system & Cost benefit analysis of the system.

Keywords: Reliability, Cost benefit analysis, Mean sojourn time, probability and availability

1.1 INTRODUCTION

Reliability is an important concept at the planning design and operation stages of various complex systems. Various authors including [6, 8, 15, 24] have studied the stochastic behavior of two-unit standby redundant system models and various measure of system effectiveness where derived by using regenerative point technique. In these papers the authors did not consider the concept of human error failure. It may be observed in electronic systems 20-30% of failure due to human error. Keeping this fact into consideration many authors [1-4, 7, 19-20, 26] have studied some standby system models taking into account the hardware and human error failure. In all these papers constant failure rates for hardware and human error have been assumed.

Recently Mahmoud [16] has studied a two-unit cold standby redundant system subject to hardware and human error failures. He derived various reliability measures for the system effectiveness.

Later on Mahmoud and Esmail [17] investigate a two-unit warm standby system subject to hardware and human error.

The aim of the present paper is to study the probabilistic analysis of the system model analysed by [17]. All repair time distributions are taken general where as all failure time distributions are exponential.

By using regenerative point technique, we analyses this system model to get various parameters interest as follows:-

(a) Steady state transition probabilities.
(b) Mean sojourn time.
(c) Reliability and mean time to system failure.
(d) Point wise and steady state availability of the system.
(e) Probability that the repairman is busy.
(f) Cost benefit analysis of the system.

1.2 MATERIALS AND METHODS

In this study we analyzed the probabilistic analysis of the system by using of the regenerative processes and have obtained expressions for the various measure of system effectiveness such as the time dependent availability, steady state availability, total fraction of busy period for the regular and expert repairman and total number of visits by the expert repairman per unit time. Using the above measures, profit was calculated. Numerical expression for steady state availability and the profit function were obtained and graphs are also drawn for various parameters involved in the system. We have compared the characteristic, availability and profit with respect to failure rate of the system, to determine when they are improved.

1.3 SYSTEM DESCRIPTION AND ASSUMPTIONS

1. The system consists of two identical units. Initially is operative and the other is kept as warm stand by.
2. Each unit has two modes normal (N) and total failure (F).
3. The operative unit can fail due to hardware failure and human error failure.
4. The stand by unit can fail due to warm stand by failure.
5. As soon as the operative unit or warm standby unit fail other unit is operative.
6. The system failure occurs when both the units fail.
7. The failure time distributions of operative unit and warm stand by unit are exponential while repair time distribution are general for three types of repairs.
8. A single repair man is always available with the system to repair a unit failed due to operation or warm stand by.
9. The repair discipline is FCFS.
10. The switching and sensing devices are perfect and instantaneous.
11. All the random variables denoting the failure times, repair times are stochastically independent.
12. The repaired unit acts as good as new.

1.4 NOTATIONS AND STAUES OF THE SYSTAM

N_o : Unit is in normal mode and operative.

N_s : Unit is in normal mode and warm standby.

F_{r_i} : Failed unit is under repair of type –i, i=1, 2.

F_{R_i} : Repair of type-i, I =1, 2 continued from the previous state.

F_{r_s} : Failed unit is under repair of warm standby.

F_{R_s} : Repair of warm stand continued from the previous state.

F_{wr_i} : Failed unit is waiting of repair of type –i, i=1, 2 if repairman is busy

F_{w_s} : Failed unit is waiting of repair of warm standby if repairman is busy.

$\alpha_1/\alpha_2/\alpha_s$: Constant failure rate of unit when it fails due to hardware failure/ human error failure/ warm standby failure.

$G_1(.), g_1(.)$: c.d.f. and p.d.f. of repair time of hardware failure.

$G_2(.), g_2(.)$: c.d.f. and p.d.f. of repair time of human error failure

$G_s(.), g_s(.)$: c.d.f. and p.d.f. of repair time of warm standby.

Thus, considering the above symbols under the assumption of the system model, the possible states of the system are as:

Up states:

$S_0 \equiv (N_o, N_s); S_1 \equiv (F_{r_1}, N_o)$

$S_2 \equiv (F_{r_2}, N_o)$; $S_3 \equiv (F_{r_s}, N_o)$

Failed states:

$S_4 \equiv (F_{R_1}, F_{wr_1})$; $S_5 \equiv (F_{R_1}, F_{wr_2})$; $S_6 \equiv (F_{R_2}, F_{wr_2})$

$S_7 \equiv (F_{R_2}, F_{wr_1})$; $S_8 \equiv (F_{R_s}, F_{wr_1})$; $S_9 \equiv (F_{R_s}, F_{wr_2})$

The epoch of the transition from states S_1 to S_4, S_1 to S_5, S_2 to S_6, S_2 to S_7, S_3 to S_8, and S_3 to $S_9$are non-regenerative while all the other entrance epoch into the states are regenerative. The possible transitions between the states along with transition rates/ transition time c.d.f. 's are shown in figure 1.1 .

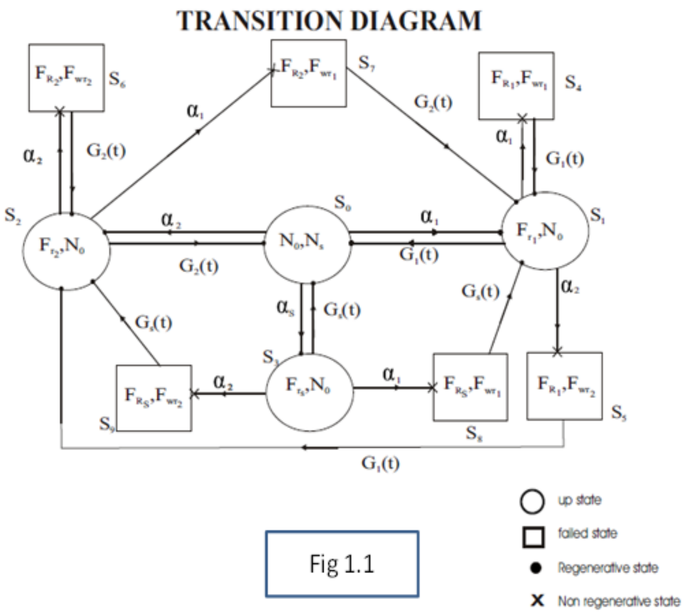

Fig 1.1

1.5 TRANSITION PROBABILITIES AND SOJOURN TIME

(a) Let Q_{ij} be the probability that the system transit from S_i to S_j on or before time t and $Q_{ij}(t)$ is the c.d.f. of transition time from regenerative state S_i to S_j So by simple probabilistic argument the non zero elements of the matrix

$Q=Q_{ij}(t)$ are

$$Q_{01}(t) = \int_0^t \alpha_1 e^{-(\alpha_1+\alpha_2+\alpha_3)u}du = \frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3}\left(1 - e^{-(\alpha_1+\alpha_2+\alpha_3)t}\right)$$

$$Q_{02}(t) = \int_0^t \alpha_2 e^{-(\alpha_1+\alpha_2+\alpha_3)u}du = \frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3}\left(1 - e^{-(\alpha_1+\alpha_2+\alpha_3)t}\right)$$

$$Q_{03}(t) = \int_0^t \alpha_3 e^{-(\alpha_1+\alpha_2+\alpha_3)u}du = \frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3}\left(1 - e^{-(\alpha_1+\alpha_2+\alpha_3)t}\right)$$

$$Q_{10}(t) = \int_0^t e^{-(\alpha_1+\alpha_2)u}\, dG_1(u)$$

$$Q_{20}(t) = \int_0^t e^{-(\alpha_1+\alpha_2)u}\, dG_2(u)$$

$$Q_{30}(t) = \int_0^t e^{-(\alpha_1+\alpha_2)u}\, dG_3(u) \qquad (1\text{-}6)$$

(b) To obtain an expression for $Q_{11}^{(4)}(t)$, we suppose that the system transit from state S_1 to S_4 during the time interval (u, u+ du) ,u $\leq$ t; the probability of this event is

$$\alpha_1 e^{-(\alpha_1+\alpha_2)u}\overline{G}_1(u)du$$

Further, suppose that the system pass from state S_4 to S_1 during the interval (v, v+ dv) in (u, t) the probability of this event is $\frac{dF(v)}{\overline{\overline{F}}(u)}$

Thus,

$$Q_{11}^{(4)} = \int_0^t \alpha_1 e^{-(\alpha_1+\alpha_2)u}\overline{G}_1(u)du \int_u^t \frac{dG_1(v)}{\overline{\overline{G}}_1(u)} \;;\; 0<u<v<t$$

$$= \alpha_1 \int_0^t dG_1(u) \int_0^v e^{-(\alpha_1+\alpha_2)u}\, du$$

$$= \frac{\alpha_1}{\alpha_1+\alpha_2}\int_0^t dG_1(v) - \frac{\alpha_1}{\alpha_1+\alpha_2}\int_0^t e^{-(\alpha_1+\alpha_2)v}\, dG_1(v)$$

$$Q_{12}^{(5)} = \int_0^t \alpha_2 e^{-(\alpha_1+\alpha_2)u}\overline{G}_1(u)du \int_u^t \frac{dG_1(v)}{\overline{\overline{G}}_1(u)}$$

$$= \alpha_2 \int_0^t dG_1(u) \int_0^v e^{-(\alpha_1+\alpha_2)u}\, du$$

$$= \frac{\alpha_2}{\alpha_1+\alpha_2}\int_0^t dG_1(v) - \frac{\alpha_2}{\alpha_1+\alpha_2}\int_0^t e^{-(\alpha_1+\alpha_2)v}\, dG_1(v)$$

$$Q_{22}^{(6)} = \int_0^t \alpha_2 e^{-(\alpha_1+\alpha_2)u}\overline{G}_2(u)du \int_u^t \frac{dG_2(v)}{\overline{\overline{G}}_2(u)}$$

$$= \frac{\alpha_2}{\alpha_1+\alpha_2}\int_0^t dG_2(v) - \frac{\alpha_2}{\alpha_1+\alpha_2}\int_0^t e^{-(\alpha_1+\alpha_2)v}\, dG_1(v)$$

$$Q_{21}^{(7)} = \int_0^t \alpha_1 e^{-(\alpha_1+\alpha_2)u}\overline{G}_2(u)du \int_u^t \frac{dG_2(v)}{\overline{\overline{G}}_2(u)}$$

$$= \frac{\alpha_1}{\alpha_1+\alpha_2}\int_0^t dG_2(v) - \frac{\alpha_1}{\alpha_1+\alpha_2}\int_0^t e^{-(\alpha_1+\alpha_2)v}\, dG_2(v)$$

$$Q_{31}^{(8)} = \int_0^t \alpha_1 e^{-(\alpha_1+\alpha_2)u}\overline{G}_3(u)du \int_u^t \frac{dG_3(v)}{\overline{\overline{G}}_3(u)}$$

$$= \frac{\alpha_1}{\alpha_1+\alpha_2}\int_0^t dG_2(v) - \frac{\alpha_1}{\alpha_1+\alpha_2}\int_0^t e^{-(\alpha_1+\alpha_2)v}\, dG_3(v)$$

$$Q_{32}^{(9)} = \int_0^t \alpha_2 e^{-(\alpha_1+\alpha_2)u}\overline{G}_3(u)du \int_u^t \frac{dG_3(v)}{\overline{\overline{G}}_3(u)}$$

$$= \frac{\alpha_2}{\alpha_1+\alpha_2}\int_0^t dG_3(v) - \frac{\alpha_2}{\alpha_1+\alpha_2}\int_0^t e^{-(\alpha_1+\alpha_2)v}\, dG_3(v) \qquad (7\text{-}12)$$

(c) Steady state transition probability: The steady state transition probability can be obtained by using result $p_{ij} = \lim_{t\to\infty} Q_{ij}(t)$ And $p_{ij}^{(k)} = \lim_{t\to\infty} Q_{ij}^{(k)}(t)$

Therefore, taking the limt $t \to \infty$ in the above (1-12) the transition probabilities we get as follows:-

$$p_{01}(t) = \int \alpha_1 e^{-(\alpha_1+\alpha_2+\alpha_3)t}dt = \frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3}$$
$$p_{02}(t) = \int \alpha_2 e^{-(\alpha_1+\alpha_2+\alpha_3)t}dt = \frac{\alpha_2}{\alpha_1+\alpha_2+\alpha_3}$$
$$p_{03}(t) = \int \alpha_3 e^{-(\alpha_1+\alpha_2+\alpha_3)t}dt = \frac{\alpha_3}{\alpha_1+\alpha_2+\alpha_3}$$
$$p_{10}(t) = \int e^{-(\alpha_1+\alpha_2)t}\, dG_1(t) = \tilde{G}_1(\alpha_1+\alpha_2)$$
$$p_{20}(t) = \int e^{-(\alpha_1+\alpha_2)t}\, dG_2(t) = \tilde{G}_2(\alpha_1+\alpha_2)$$
$$p_{30}(t) = \int e^{-(\alpha_1+\alpha_2)t}\, dG_3(t) = \tilde{G}_3(\alpha_1+\alpha_2)$$
$$p_{11}^{(4)} = \frac{\alpha_1}{\alpha_1+\alpha_2}\int dG_1(t) - \frac{\alpha_1}{\alpha_1+\alpha_2}\int e^{-(\alpha_1+\alpha_2)t}\, dG_1(t)$$
$$= \frac{\alpha_1}{\alpha_1+\alpha_2}[1-\tilde{G}_1(\alpha_1+\alpha_2)]$$
$$p_{12}^{(5)} = \frac{\alpha_2}{\alpha_1+\alpha_2}\int dG_1(t) - \frac{\alpha_2}{\alpha_1+\alpha_2}\int e^{-(\alpha_1+\alpha_2)t}\, dG_1(t)$$
$$= \frac{\alpha_2}{\alpha_1+\alpha_2}[1-\tilde{G}_1(\alpha_1+\alpha_2)]$$
$$p_{22}^{(6)} = \frac{\alpha_2}{\alpha_1+\alpha_2}\int dG_2(t) - \frac{\alpha_2}{\alpha_1+\alpha_2}\int e^{-(\alpha_1+\alpha_2)t}\, dG_2(t)$$
$$= \frac{\alpha_2}{\alpha_1+\alpha_2}[1-\tilde{G}_2(\alpha_1+\alpha_2)]$$
$$p_{21}^{(7)} = \frac{\alpha_1}{\alpha_1+\alpha_2}\int dG_2(t) - \frac{\alpha_1}{\alpha_1+\alpha_2}\int e^{-(\alpha_1+\alpha_2)t}\, dG_2(t)$$
$$= \frac{\alpha_1}{\alpha_1+\alpha_2}[1-\tilde{G}_2(\alpha_1+\alpha_2)]$$
$$p_{31}^{(8)} = \frac{\alpha_1}{\alpha_1+\alpha_2}\int dG_3(t) - \frac{\alpha_1}{\alpha_1+\alpha_2}\int e^{-(\alpha_1+\alpha_2)t}\, dG_3(t)$$
$$= \frac{\alpha_1}{\alpha_1+\alpha_2}[1-\tilde{G}_3(\alpha_1+\alpha_2)]$$
$$p_{32}^{(9)} = \frac{\alpha_2}{\alpha_1+\alpha_2}\int dG_3(t) - \frac{\alpha_2}{\alpha_1+\alpha_2}\int e^{-(\alpha_1+\alpha_2)t}\, dG_3(t)$$
$$= \frac{\alpha_2}{\alpha_1+\alpha_2}[1-\tilde{G}_3(\alpha_1+\alpha_2)] \qquad (13\text{-}24)$$

It can be easily verified that

$$p_{01} + p_{02} + p_{03} = 1$$
$$p_{10} + p_{11}^{(4)} + p_{12}^{(5)} = 1$$
$$p_{20} + p_{22}^{(6)} + p_{21}^{(7)} = 1$$
$$p_{30} + p_{31}^{(8)} + p_{32}^{(9)} = 1 \qquad (25\text{-}28)$$

(d) Mean Sojourn Time: Mean sojourn time Ψ_i in state S_i is defined as the expected time for which the system stays in state S_i before transiting to any other state.

In particular, to calculate the mean sojourn time Ψ_0 in the state S_0, we observe that so long as the system is in state S_0 ,there is no transition to state S_1, S_2 and S_3. Hence if T_0 denotes the sojourn time in S_0

Then,

$$\Psi_0 = \int P[\,T_0 > t\,]dt$$
$$= \int e^{-(\alpha_1+\alpha_2+\alpha_3)t}dt = \frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3}$$
$$\Psi_1 = \int e^{-(\alpha_1+\alpha_2)t}\overline{G}_1(t)dt = \int e^{-(\alpha_1+\alpha_2)t}dt - \int e^{-(\alpha_1+\alpha_2)t}G_1\,dt$$

$$= \frac{1}{\alpha_1+\alpha_2} - G_1^*(\alpha_1+\alpha_2) = \frac{1-\tilde{G}_1(\alpha_1+\alpha_2)}{\alpha_1+\alpha_2}$$

$$\Psi_2 = \int e^{-(\alpha_1+\alpha_2)t}\overline{G}_2(t)dt = \int e^{-(\alpha_1+\alpha_2)t}dt - \int e^{-(\alpha_1+\alpha_2)t}G_2 dt$$

$$= \frac{1}{\alpha_1+\alpha_2} - G_2^*(\alpha_1+\alpha_2) = \frac{1-\tilde{G}_2(\alpha_1+\alpha_2)}{\alpha_1+\alpha_2}$$

$$\Psi_3 = \int e^{-(\alpha_1+\alpha_2)t}\overline{G}_3(t)dt = \int e^{-(\alpha_1+\alpha_2)t}dt - \int e^{-(\alpha_1+\alpha_2)t}G_3 dt$$

$$= \frac{1}{\alpha_1+\alpha_2} - G_3^*(\alpha_1+\alpha_2) = \frac{1-\tilde{G}_3(\alpha_1+\alpha_2)}{\alpha_1+\alpha_2}$$

$$\Psi_4 = \int \overline{G}_1(t)dt = m_1 = \Psi_5$$

$$\Psi_6 = \int \overline{G}_2(t)dt = m_2 = \Psi_7$$

$$\Psi_8 = \int \overline{G}_3(t)dt = m_3 = \Psi_9 \qquad (29\text{-}35)$$

(e) Conditional Mean Sojourn Time: To obtained the conditional mean sojourn time in state S_i , given that the system is to transit to state S_j ,we define

m_{ij} = conditional mean sojourn time in state S_i /the system is to transit to state S_j

$$= \int t q_{ij}(t)dt = \int t dQ_{ij}(t) = E[T_{ij}]$$

$$= -\frac{d}{ds}\tilde{Q}_{ij}(s)/_{s=0} = -\frac{d}{ds}q_{ij}^*(s)/_{s=0}$$

In particular,

$$m_{01} = \int t\alpha_1 e^{-(\alpha_1+\alpha_2+\alpha_3)t}dt = \frac{\alpha_1}{(\alpha_1+\alpha_2+\alpha_3)^2}$$

$$m_{02} = \int t\alpha_2 e^{-(\alpha_1+\alpha_2+\alpha_3)t}dt = \frac{\alpha_2}{(\alpha_1+\alpha_2+\alpha_3)^2}$$

$$m_{03} = \int t\alpha_3 e^{-(\alpha_1+\alpha_2+\alpha_3)t}dt = \frac{\alpha_3}{(\alpha_1+\alpha_2+\alpha_3)^2}$$

$$m_{10} = \int t e^{-(\alpha_1+\alpha_2)t}\, dG_1(t)$$

$$m_{20} = \int t e^{-(\alpha_1+\alpha_2)t}\, dG_2(t)$$

$$m_{30} = \int t e^{-(\alpha_1+\alpha_2)t}\, dG_3(t)$$

$$m_{11}^{(4)} = \frac{\alpha_1}{\alpha_1+\alpha_2}\int t dG_1(t) - \frac{\alpha_1}{\alpha_1+\alpha_2}\int t e^{-(\alpha_1+\alpha_2)t}\, dG_1(t)$$

$$m_{12}^{(5)} = \frac{\alpha_2}{\alpha_1+\alpha_2}\int t dG_1(t) - \frac{\alpha_2}{\alpha_1+\alpha_2}\int t e^{-(\alpha_1+\alpha_2)t}\, dG_1(t)$$

$$m_{22}^{(6)} = \frac{\alpha_2}{\alpha_1+\alpha_2}\int t dG_2(t) - \frac{\alpha_2}{\alpha_1+\alpha_2}\int t e^{-(\alpha_1+\alpha_2)t}\, dG_2(t)$$

$$m_{21}^{(7)} = \frac{\alpha_1}{\alpha_1+\alpha_2}\int t dG_2(t) - \frac{\alpha_1}{\alpha_1+\alpha_2}\int t e^{-(\alpha_1+\alpha_2)t}\, dG_2(t)$$

$$m_{31}^{(8)} = \frac{\alpha_1}{\alpha_1+\alpha_2}\int t dG_3(t) - \frac{\alpha_1}{\alpha_1+\alpha_2}\int t e^{-(\alpha_1+\alpha_2)t}\, dG_3(t)$$

$$m = \frac{\alpha_2}{\alpha_1+\alpha_2}\int t dG_3(t) - \frac{\alpha_2}{\alpha_1+\alpha_2}\int t e^{-(\alpha_1+\alpha_2)t}\, dG_3(t) \qquad (36\text{-}47)$$

It can be easily verified that

$$m + m_{02} + m_{03} = \frac{1}{\alpha_1+\alpha_2+\alpha_3} = \Psi_0$$

$$m_{10} + m_{11}^{(4)} + m_{12}^{(5)} = \int t dG_1(t) = \int \overline{G}_1(t)dt = m_1$$

$$m_{20} + m_{22}^{(6)} + m_{21}^{(7)} = \int t dG_2(t) = \int \overline{G}_2(t)dt = m_2$$

$$m_{30} + m_{31}^{(8)} + m_{32}^{(9)} = \int t dG_3(t) = \int \overline{G}_3(t)dt = m_3 \qquad (48\text{-}51)$$

1.6 RELIABILITY OF THE SYSTEM AND MTSF

Let the r.v. T_i denote the time to system failure when the system starts from state S_i, then the system reliability according to its definition is given by

$$R_i(t) = P[T_i > t]$$

To determine it, we regard the failed states of the system S_4, S_5, S_6, S_7, S_8 and S_9, as absorbing states.

By probabilistic arguments, we have

$$R_0(t) = Z_0(t) + q_{01}(t) © R_1(t) + q_{02}(t) © R_2(t) + q_{03}(t) © R_3(t)$$
$$R_1(t) = Z_1(t) + q_{10}(t) © R_0(t)$$
$$R_2(t) = Z_2(t) + q_{20}(t) © R_0(t)$$
$$R_3(t) = Z_3(t) + q_{30}(t) © R_0(t) \qquad (52\text{-}55)$$

Where,

$$Z_0(t) = e^{-(a_1+a_2+a_3)t}$$
$$Z_1(t) = e^{-(a_1+a_2)t}\bar{G}_1(t)$$
$$Z_2(t) = e^{-(a_1+a_2)t}\bar{G}_2(t)$$
$$Z_3(t) = e^{-(a_1+a_2)t}\bar{G}_3(t)$$

As an illustration, $R_0(t)$ is the sum of the following contingencies:-

(i) The system remain up in state S_0 without making any transition to any other state up to time t. The probability of this contingency is
$e^{-(a_1+a_2+a_3)t} = Z_0(t)$, (say)

(ii) System first enters into the state S_i (i=1,2,3) from the state S_0 during (u, u+du) u<t, and then starting from S_i (i=1,2,3) it remain up continuously during remaining time (t-u). The probability of this contingency is

$$\int_0^t q_{01}(u)R_1(t-u) + \int_0^t q_{02}(u)R_2(t-u) + \int_0^t q_{03}(u)R_3(t-u)$$
$$= q_{01}(t) © R_1(t) + q_{02}(t) © R_2(t) + q_{03}(t) © R_3(t)$$

Taking the Laplace Transform of relation (52-55). The solution for $R_i^*(s)$ can be written in the matrix form as follow :-

$$\begin{vmatrix} R_0^* \\ R_1^* \\ R_2^* \\ R_3^* \end{vmatrix} = \begin{vmatrix} 1 & -q_{01}^* & -q_{02}^* & -q_{03}^* \\ -q_{10}^* & 1 & 0 & 0 \\ -q_{20}^* & 0 & 1 & 0 \\ q_{30}^* & 0 & 0 & 1 \end{vmatrix}^{-1} \begin{vmatrix} Z_0^* \\ Z_1^* \\ Z_2^* \\ Z_3^* \end{vmatrix} \qquad (56)$$

For brevity, the argument 's' is omitted from $q_{ij}^*(s)$, $Z_i^*(s)$ and $R_i^*(s)$.

Computing the elements of the inverse matrix. We have

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} \qquad (57)$$

Where,

$$N_1(s) = Z_0^* + q_{01}^* Z_1^* + q_{02}^* Z_2^* + q_{03}^* Z_3^* \qquad (58)$$

And

$$D_1(s) = 1 - q_{01}^* q_{10}^* - q_{02}^* q_{20}^* - q_{03}^* q_{30}^* \qquad (59)$$

Taking the inverse Laplace Transform of (57), one can get the reliability of the system when the system start from state S_0.

To get the MTSF, we have the following well-known formula:

$$E(T_0) = \int_0^\infty R_0(t)\, dt = \lim_{s\to 0} R_0^*(s) = \frac{N_1(0)}{D_1(0)} \qquad (60)$$

To obtain $N_1(0)$ and $D_1(0)$. We use the result

$Z_i^*(0) = \Psi_1$ and $q_{ij}^*(0) = P_{ij}$

Thus,

$$N_1(0) = \Psi_0 + p_{01}\Psi_1 + p_{02}\Psi_2 + p_{03}\Psi_3 \quad (61)$$

And

$$D_1(0) = 1 - p_{01}p_{10} - p_{02}p_{20} - p_{03}p_{30} \quad (62)$$

Hence

$$E(T_0) = (\Psi_0 + p_{01}\Psi_1 + p_{02}\Psi_2 + p_{03}\Psi_3)/(1 - p_{01}p_{10} - p_{02}p_{20} - p_{03}p_{30}) \quad (63)$$

1.7 AVAILABILITY ANALYSIS

Let $A_i(t)$ be the probability that the system is up at epoch t when initially it start from state S_i.

To obtain $A_0(t)$ we observe all possible exists from S_0. So that by total Probability law, $A_0(t)$ is the sum of the following contingencies:

(i) the system continuous to be up in the state S_0 until epoch t. The probability of this event is $e^{-(a_1+a_2+a_3)t} = Z_0(t)$

(ii) The system transit from state S_0 to S_i (i=1, 2, 3) during time (u, u + du) ; 0<u<t and then starting from S_i (i=1, 2, 3), it is observed to be up at epoch t and the probability of this event is

$$\int_0^t q_{01}(u)\,du A_1(t-u) + \int_0^t q_{02}(u)\,du A_2(t-u) + \int_0^t q_{03}(u)\,du A_3(t-u)$$

$$= q_{01}(t) © A_1(t) + q_{02}(t) © A_2(t) + q_{03}(t) © A_3(t)$$

Thus

$$A_0(t) = Z_0(t) + q_{01}(t) © A_1(t) + q_{02}(t) © A_2(t) + q_{03}(t) © A_3(t)$$

Similarly,

$$A_1(t) = Z_1(t) + q_{10}(t) © A_0(t) + q_{11}^{(4)}(t) © A_1(t) + q_{12}^{(5)}(t) © (t)$$

$$A_2(t) = Z_2(t) + q_{20}(t) © A_0(t) + q_{22}^{(6)}(t) © A_2(t) + q_{21}^{(7)}(t) © A_1(t)$$

$$A_3(t) = Z_3(t) + q_{30}(t) © A_0(t) + q_{31}^{(8)}(t) © A_1(t) + q_{32}^{(9)}(t) © A_2(t) \quad (64\text{-}67)$$

Taking the L. T. of relations (69-73), the resulting set of algebraic equations is as follows:

$$A_0^*(s) = Z_0^*(s) + q_{01}^*(s)A_1^*(s) + q_{02}^*(s)A_2^*(s) + q_{03}^*(s)A_3^*(s)$$

$$A_1^*(s) = Z_1^*(s) + q_{10}^*(s)A_0^*(s) + q_{11}^{(4)*}(s)A_1^*(s) + q_{12}^{(5)*}(s)A_2^*(s)$$

$$A_2^*(s) = Z_2^*(s) + q_{20}^*(s)A_0^*(s) + q_{22}^{(6)*}(s)A_2^*(s) + q_{21}^{(7)*}(s)A_1^*(s)$$

$$A_3^*(s) = Z_3^*(s) + q_{30}^*(s)A_0^*(s) + q_{31}^{(8)*}(s)A_1^*(s) + q_{32}^{(9)*}(s)A_2^*(s) \quad (68\text{-}71)$$

Now this set of equations (68-71) can be written in matrix form as:

$$\begin{vmatrix} A_0^* \\ A_1^* \\ A_2^* \\ A_3^* \end{vmatrix} = \begin{vmatrix} 1 & -q_{01}^* & -q_{02}^* & -q_{03}^* \\ -q_{10}^* & 1-q_{11}^{(4)*} & -q_{12}^{(5)*} & 0 \\ -q_{20}^* & -q_{21}^{(7)*} & 1-q_{22}^{(6)*} & 0 \\ -q_{30}^* & -q_{31}^{(8)*} & -q_{32}^{(9)*} & 1 \end{vmatrix}^{-1} \begin{vmatrix} Z_0^* \\ Z_1^* \\ Z_2^* \\ Z_3^* \end{vmatrix} \quad (72)$$

Solve the above matrix for $A_0^*(s)$, we get

$$A_0^*(s) = \frac{N_2(s)}{D_2(s)} \quad (73)$$

Where,

$$N_2(s) = Z_0^*\left[\left(1 - q_{11}^{(4)*}\right)\left(1 - q_{22}^{(6)*}\right) - q_{12}^{(5)*} q_{21}^{(7)*}\right]$$
$$+ Z_1^*[q_{01}^*\left(1 - q_{22}^{(6)*}\right) + q_{02}^* q_{21}^{(7)*} + q_{03}^*\{q_{21}^{(7)*} q_{32}^{(9)*} + q_{31}^{(8)*}\left(1 - q_{22}^{(6)*}\right)\}]$$

$$
\begin{aligned}
&+Z_2^*[q_{01}^* q_{12}^{(5)*} + q_{02}^*\left(1 - q_{11}^{(4)*}\right) + q_{03}^*\{\left(1 - q_{11}^{(4)*}\right)q_{32}^{(9)*} + q_{31}^{(8)*} q_{12}^{(5)*}\}] \\
&+Z_3^* q_{03}^*[\left(1 - q_{11}^{(4)*}\right)\left(1 - q_{22}^{(6)*}\right) - q_{21}^{(7)*} q_{12}^{(5)*}] \qquad (74)
\end{aligned}
$$

$$
\begin{aligned}
D_2(s) = &\left(1 - q_{11}^{(4)*}\right)\left(1 - q_{22}^{(6)*}\right) - q_{12}^{(5)*} q_{21}^{(7)*} \\
&- q_{10}^*[q_{01}^*\left(1 - q_{22}^{(6)*}\right) + q_{02}^* q_{21}^{(7)*} + q_{03}^*\{q_{21}^{(7)*} q_{32}^{(9)*} + q_{31}^{(8)*}\left(1 - q_{22}^{(6)*}\right)\}] \\
&- q_{20}^*[q_{01}^* q_{12}^{(5)*} + q_{02}^*\left(1 - q_{11}^{(4)*}\right) + q_{03}^*\{\left(1 - q_{11}^{(4)*}\right)q_{32}^{(9)*} + q_{31}^{(8)*} q_{12}^{(5)*}\}] \\
&- q_{30}^* q_{03}^*[\left(1 - q_{11}^{(4)*}\right)\left(1 - q_{22}^{(6)*}\right) - q_{21}^{(7)*} q_{12}^{(5)*}] \qquad (75)
\end{aligned}
$$

Now to obtain the steady state probability that the system will be operative when initially it starts from S_i , we proceed as follows by using the result

$Z_i^*(0) = \int_0^\infty Z_i(t)dt = \Psi_i$ And $q_{ij}^*(0) = p_{ij}$

Therefore,

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} sA_0^*(s) = \lim_{s\to 0} s\frac{N_2(s)}{D_2(s)} \qquad (76)$$

Now,

$$
\begin{aligned}
D_2(0) = &\left(1 - p_{11}^{(4)}\right)\left(1 - p_{22}^{(6)}\right) - p_{12}^{(5)} p_{21}^{(7)} \\
&- p_{10}[p_{01}\left(1 - p_{22}^{(6)}\right) + p_{02} p_{21}^{(7)} + p_{03}\{p_{21}^{(7)} p_{32}^{(9)} + p_{31}^{(8)}\left(1 - p_{22}^{(6)}\right)\}] \\
&- p_{20}[p_{01} p_{12}^{(5)} + p_{02}\left(1 - p_{11}^{(4)}\right) + p_{03}\{\left(1 - p_{11}^{(4)}\right)p_{32}^{(9)} + p_{31}^{(8)} p_{12}^{(5)}\}] \\
&- p_{30} p_{03}[\left(1 - p_{11}^{(4)}\right)\left(1 - p_{22}^{(6)}\right) - p_{21}^{(7)} p_{12}^{(5)}] \\
&= 0
\end{aligned}
$$

Since, $D_2(0) = 0$. Therefore,

$$A_0 = \frac{N_2(0)}{D_2'(0)} = \frac{N_2}{D_2} \qquad \text{(By using L. Hospital rule)} \qquad (77)$$

Where,

$$
\begin{aligned}
N_2 = &\Psi_0[\left(1 - p_{11}^{(4)}\right)\left(1 - p_{22}^{(6)}\right) - p_{12}^{(5)} p_{21}^{(7)}] \\
&+ \Psi_1[p_{01}\left(1 - p_{22}^{(6)}\right) + p_{02} p_{21}^{(7)} + p_{03}\{p_{21}^{(7)} p_{32}^{(9)} + p_{31}^{(8)}\left(1 - p_{22}^{(6)}\right)\}] \\
&+ \Psi_2[p_{01} p_{12}^{(5)} + p_{02}\left(1 - p_{11}^{(4)}\right) + p_{03}\{\left(1 - p_{11}^{(4)}\right)p_{32}^{(9)} + p_{31}^{(8)} p_{12}^{(5)}\}] \\
&+ \Psi_3 p_{03}[\left(1 - p_{11}^{(4)}\right)\left(1 - p_{22}^{(6)}\right) - p_{21}^{(7)} p_{12}^{(5)}] \qquad (78)
\end{aligned}
$$

Now, to obtain D_2 , we collect the coefficient of $m_{ij} = -q_{ij}^{*\prime}(0)$ in $D_2'(0)$ for various value of i and j as follows:

(1) coefficient of $m_{01} = p_{10}\left(1 - p_{22}^{(6)}\right) + p_{20} p_{12}^{(5)}$

(2) coefficient of $m_{02} = p_{10} p_{21}^{(7)} + p_{20}\left(1 - p_{11}^{(4)}\right)$

$$
\begin{aligned}
&= p_{10}\left(1 - p_{20} - p_{22}^{(6)}\right) + p_{20}\left(p_{10} + p_{12}^{(5)}\right) \\
&= p_{10}\left(1 - p_{22}^{(6)}\right) + p_{20} p_{12}^{(5)}
\end{aligned}
$$

(3) coefficient of $m_{03} = p_{10}\left[p_{21}^{(7)}p_{32}^{(9)} + p_{31}^{(8)}\left(1-p_{22}^{(6)}\right)\right] + p_{20}[\left(1-p_{11}^{(4)}\right)p_{32}^{(9)} + p_{31}^{(8)}p_{12}^{(5)}]$

$$+p_{30}[\left(1-p_{11}^{(4)}\right)\left(1-p_{22}^{(6)}\right) - p_{21}^{(7)}p_{12}^{(5)}]$$

$$= p_{10}\left(1-p_{22}^{(6)}\right)p_{32}^{(9)} - p_{10}p_{20}p_{32}^{(9)} + p_{10}p_{31}^{(8)}\left(1-p_{22}^{(6)}\right) + p_{20}p_{12}^{(5)}p_{32}^{(9)}$$

$$+p_{20}p_{12}^{(5)}p_{31}^{(8)} + p_{30}\left(1-p_{22}^{(6)}\right) - p_{30}p_{11}^{(4)}\left(1-p_{22}^{(6)}\right) - p_{30}p_{12}^{(5)}p_{21}^{(7)}$$

$$= p_{10}\left(1-p_{22}^{(6)}\right) - p_{10}\left(1-p_{22}^{(6)}\right)p_{20} - p_{01}\left(1-p_{22}^{(6)}\right)p_{31}^{(8)}$$

$$+p_{01}\left(1-p_{22}^{(6)}\right)p_{31}^{(8)} + p_{20}p_{12}^{(5)}p_{32}^{(9)} + p_{20}p_{12}^{(5)}p_{31}^{(8)} + p_{30}\left(1-p_{22}^{(6)}\right)$$

$$-p_{30}p_{12}^{(5)}p_{21}^{(7)} - p_{30}\left(1-p_{22}^{(6)}\right) + p_{10}\left(1-p_{22}^{(6)}\right)p_{30} +$$

$$p_{12}^{(5)}\left(1- p_{22}^{(6)}\right)p_{30}$$

$$= p_{10}\left(1-p_{22}^{(6)}\right) + p_{20}p_{12}^{(5)} - p_{20}p_{12}^{(5)}p_{30} - p_{30}p_{12}^{(5)}p_{21}^{(7)} + p_{30}p_{20}p_{12}^{(5)}$$

$$+p_{30}p_{12}^{(5)}p_{21}^{(7)}$$

$$= p_{10}\left(1-p_{22}^{(6)}\right) + p_{20}p_{12}^{(5)}$$

(4) coefficient of $m_{10} = p_{01}\left(1-p_{22}^{(6)}\right) + p_{02}p_{21}^{(7)} + p_{03}p_{21}^{(7)}p_{32}^{(9)} + p_{03}\left(1-p_{22}^{(6)}\right)p_{31}^{(8)}$

(5) coefficient of $m_{11}^{(4)} = \left(1-p_{22}^{(6)}\right) - p_{02}p_{20} - p_{03}p_{20}p_{32}^{(9)} - p_{03}p_{30}\left(1-p_{22}^{(6)}\right)$

$$= \left(1-p_{22}^{(6)}\right) - p_{02}\left(1-p_{22}^{(6)}\right) + p_{02}p_{21}^{(7)} + p_{03}p_{32}^{(9)}\left(1-p_{22}^{(6)}\right)$$

$$+p_{02}p_{21}^{(7)}p_{32}^{(9)} - p_{03}\left(1-p_{22}^{(6)}\right) - p_{03}p_{31}^{(8)}\left(1-p_{22}^{(6)}\right)$$

$$+p_{32}^{(9)}p_{03}\left(1-p_{22}^{(6)}\right)$$

$$= \left(1-p_{22}^{(6)}\right) - \left(1-p_{22}^{(6)}\right) + p_{01}\left(1-p_{22}^{(6)}\right) + p_{03}\left(1-p_{22}^{(6)}\right) +$$

$$+p_{02}p_{21}^{(7)} - p_{32}^{(9)}p_{03}\left(1-p_{22}^{(6)}\right) + p_{03}p_{21}^{(7)}p_{32}^{(9)} - p_{03}\left(1-p_{22}^{(6)}\right) +$$

$$p_{03}p_{31}^{(8)}\left(1-p_{22}^{(6)}\right) + p_{03}p_{32}^{(9)}\left(1-p_{22}^{(6)}\right)$$

$$= p_{01}\left(1-p_{22}^{(6)}\right) + p_{02}p_{21}^{(7)} + p_{03}p_{21}^{(7)}p_{32}^{(9)} + p_{03}\left(1-p_{22}^{(6)}\right)p_{31}^{(8)}$$

(6) coefficient of $m_{12}^{(5)} = p_{21}^{(7)} + p_{01}p_{20} + p_{03}p_{20}p_{31}^{(8)} - p_{03}p_{30}p_{21}^{(7)}$

$$= p_{21}^{(7)} + p_{01}\left(1-p_{22}^{(6)}\right) - p_{01}p_{21}^{(7)} + p_{03}\left(1-p_{22}^{(6)}\right)p_{31}^{(8)} - p_{03}p_{21}^{(7)}p_{31}^{(8)}$$

$$-p_{03}p_{30}p_{21}^{(7)}$$

$$= p_{01}\left(1-p_{22}^{(6)}\right) + p_{03}\left(1-p_{22}^{(6)}\right)p_{31}^{(8)} + p_{03}p_{21}^{(7)}p_{32}^{(9)} - p_{03}p_{21}^{(7)}$$

$$+p_{03}p_{30}p_{21}^{(7)} + p_{21}^{(7)} - p_{01}p_{21}^{(7)} - p_{03}p_{30}p_{21}^{(7)}$$

$$= p_{01}\left(1-p_{22}^{(6)}\right) + p_{03}\left(1-p_{22}^{(6)}\right)p_{31}^{(8)} + p_{03}p_{21}^{(7)}p_{32}^{(9)} - p_{21}^{(7)} + p_{02}p_{21}^{(7)}$$

$$+p_{01}p_{21}^{(7)} + p_{21}^{(7)} - p_{01}p_{21}^{(7)} - p_{03}p_{30}p_{21}^{(7)} + p_{03}p_{30}p_{21}^{(7)}$$

$$= p_{01}\left(1-p_{22}^{(6)}\right) + p_{02}p_{21}^{(7)} + p_{03}p_{21}^{(7)}p_{32}^{(9)} + p_{03}\left(1-p_{22}^{(6)}\right)p_{31}^{(8)}$$

(7) coefficient of $m_{20} = p_{01}p_{12}^{(5)} + p_{02}\left(1-p_{11}^{(4)}\right) + p_{03}p_{32}^{(9)}\left(1-p_{11}^{(4)}\right) + p_{03}p_{12}^{(5)}p_{31}^{(8)}$

(8) coefficient of $m_{21}^{(7)} = p_{12}^{(5)} + p_{02}p_{10} + p_{03}p_{10}p_{32}^{(9)} - p_{03}p_{30}p_{12}^{(5)}$

$$= p_{02}\left(1-p_{11}^{(4)}\right) - p_{02}p_{12}^{(5)} + p_{12}^{(5)} + p_{03}p_{32}^{(9)}\left(1-p_{11}^{(4)}\right) - p_{03}p_{12}^{(5)}p_{32}^{(9)} - p_{03}p_{30}p_{12}^{(5)}$$

$$= p_{02}\left(1-p_{11}^{(4)}\right) + p_{03}p_{32}^{(9)}\left(1-p_{11}^{(4)}\right) + p_{01}p_{12}^{(5)} - p_{12}^{(5)} + p_{03}p_{12}^{(5)} + p_{12}^{(5)} - p_{03}p_{12}^{(5)} + p_{03}p_{12}^{(5)}p_{31}^{(8)} - p_{03}p_{30}p_{12}^{(5)} + p_{03}p_{30}p_{12}^{(5)}$$

$$= p_{01}p_{12}^{(5)} + p_{02}\left(1-p_{11}^{(4)}\right) + p_{03}p_{32}^{(9)}\left(1-p_{11}^{(4)}\right) + p_{03}p_{12}^{(5)}p_{31}^{(8)}$$

(9) coefficient of $m_{22}^{(6)} = \left(1-p_{11}^{(4)}\right) - p_{01}p_{10} - p_{03}p_{10}p_{31}^{(8)} - p_{03}p_{30}\left(1-p_{11}^{(4)}\right)$

$$= \left(1-p_{11}^{(4)}\right) - p_{01}\left(1-p_{11}^{(4)}\right) + p_{01}p_{12}^{(5)} - p_{03}p_{31}^{(8)}\left(1-p_{11}^{(4)}\right) + p_{03}p_{12}^{(5)}p_{31}^{(8)} - p_{03}\left(1-p_{11}^{(4)}\right) + p_{03}p_{31}^{(8)}\left(1-p_{11}^{(4)}\right) + p_{03}p_{32}^{(9)}\left(1-p_{11}^{(4)}\right)$$

$$= p_{01}p_{12}^{(5)} + p_{03}p_{12}^{(5)}p_{31}^{(8)} + p_{03}p_{32}^{(9)}\left(1-p_{11}^{(4)}\right) + \left(1-p_{11}^{(4)}\right) - \left(1-p_{11}^{(4)}\right) + p_{02}\left(1-p_{11}^{(4)}\right) + p_{03}\left(1-p_{11}^{(4)}\right) - p_{03}\left(1-p_{11}^{(4)}\right)$$

$$= p_{01}p_{12}^{(5)} + p_{02}\left(1-p_{11}^{(4)}\right) + p_{03}p_{32}^{(9)}\left(1-p_{11}^{(4)}\right) + p_{03}p_{12}^{(5)}p_{31}^{(8)}$$

(10) coefficient of $m_{30} = p_{03}\left(1-p_{11}^{(4)}\right)\left(1-p_{22}^{(6)}\right) - p_{03}p_{12}^{(5)}p_{21}^{(7)}$

$$= p_{03}\left(1-p_{22}^{(6)}\right) - p_{03}p_{11}^{(4)}\left(1-p_{22}^{(6)}\right) - p_{03}\left(1-p_{22}^{(6)}\right)p_{12}^{(5)} + p_{03}p_{20}p_{12}^{(5)}$$

$$= p_{03}p_{20}p_{12}^{(5)} + p_{03}\left(1-p_{22}^{(6)}\right) - p_{03}\left(1-p_{22}^{(6)}\right) + p_{03}p_{12}^{(5)}\left(1-p_{22}^{(6)}\right) + p_{03}p_{10}\left(1-p_{22}^{(6)}\right) - p_{03}\left(1-p_{22}^{(6)}\right)p_{12}^{(5)}$$

$$= p_{03}p_{10}\left(1-p_{22}^{(6)}\right) + p_{03}p_{20}p_{12}^{(5)}$$

(11) coefficient of $m_{31}^{(8)} - p_{03}p_{10}\left(1-p_{22}^{(6)}\right) + p_{03}p_{20}p_{12}^{(5)}$

(12) coefficient of $m_{32}^{(9)} = p_{03}p_{10}p_{21}^{(7)} + p_{03}p_{20}\left(1-p_{11}^{(4)}\right)$

$$= p_{03}p_{10}\left(1-p_{22}^{(6)}\right) - p_{03}p_{10}p_{20} + p_{03}p_{20}p_{12}^{(5)} + p_{03}p_{10}p_{20}$$

$$= p_{03}p_{10}\left(1-p_{22}^{(6)}\right) + p_{03}p_{20}p_{12}^{(5)}$$

Therefore, we have

$$D_2 = \left[p_{10}\left(1-p_{22}^{(6)}\right) + p_{20}p_{12}^{(5)}\right](m_{01} + m_{02} + m_{03}) + \left[p_{01}\left(1-p_{22}^{(6)}\right) + p_{02}p_{21}^{(7)} + p_{03}p_{21}^{(7)}p_{32}^{(9)} + p_{03}\left(1-p_{22}^{(6)}\right)p_{31}^{(8)}\right]\left(m_{10} + m_{11}^{(4)} + m_{12}^{(5)}\right) + \left[p_{01}p_{12}^{(5)} + p_{02}\left(1-p_{11}^{(4)}\right) + p_{03}p_{32}^{(9)}\left(1-p_{11}^{(4)}\right) + p_{03}p_{12}^{(5)}p_{31}^{(8)}\right]\left(m_{20} + m_{22}^{(6)} + m_{21}^{(7)}\right) + \left[p_{03}p_{10}\left(1-p_{22}^{(6)}\right) + p_{03}p_{20}p_{12}^{(5)}\right]\left(m_{30} + m_{31}^{(8)} + m_{32}^{(9)}\right)$$

Using the relation $\sum_j m_{ij} = \Psi_i$, we get

$$D_2 = \left[p_{10}\left(1-p_{22}^{(6)}\right)+p_{20}p_{12}^{(5)}\right]\Psi_0 + \left[p_{01}\left(1-p_{22}^{(6)}\right)+p_{02}p_{21}^{(7)}+p_{03}p_{21}^{(7)}p_{32}^{(9)}+ p_{03}\left(1-p_{22}^{(6)}\right)p_{31}^{(8)}\right]m_1+[p_{01}p_{12}^{(5)}+ p_{02}\left(1-p_{11}^{(4)}\right)+p_{03}p_{32}^{(9)}\left(1-p_{11}^{(4)}\right)+ p_{03}p_{12}^{(5)}p_{31}^{(8)}]m_2+[p_{03}p_{10}\left(1-p_{22}^{(6)}\right)+p_{03}p_{20}p_{12}^{(5)}]m_3 \quad (79)$$

Thus, on using (78) and (79) in (77), we obtain the steady state probability that the system is up.

The mean up time of the system during (0, t) is

$$\mu_{up}(t) = \int_0^t A_0(u)du \quad (80)$$

So that,

$$\mu_{up}^*(s) = \frac{A_0^*(s)}{s} \quad (81)$$

1.8 Busy Period Analysis

(a) Busy period due to Hardware failure of repair: Let $B_i^1(t)$ be the probability that the repairman is busy in the repair of failed unit, which is fail due to hardware. Using elementary probabilistic arguments yield the following recursive relations.

$$B_0^1(t) = q_{01}(t) © B_1^1(t) + q_{02}(t) © B_2^1(t) + q_{03}(t) © B_3^1(t)$$
$$B_1^1(t) = Z_1(t) + q_{14}(t) © Z_4(t) + q_{15}(t) © Z_5(t) + q_{10}(t) © B_0^1(t) + q_{12}^{(5)}(t) © B_2^1(t) + q_{11}^{(4)}(t) © B_1^1(t)$$
$$B_2^1(t) = q_{20}(t) © B_0^1(t) + q_{21}^{(7)}(t) © B_1^1(t) + q_{22}^{(6)}(t) © B_2^1(t)$$
$$B_3^1(t) = q_{30}(t) © B_0^1(t) + q_{31}^{(8)}(t) © B_1^1(t) + q_{32}^{(9)}(t) © B_2^1(t) \quad (82\text{-}85)$$

As an illustration B_i^1 is the sum of the following mutually exclusive contingencies:

The system transits from S_j to S_i (i=1, 2, 3) during time (u, u+ du); 0<u<t and then starting from repairman is observed to be busy at epoch t. The probability of this event is

$$\int_0^t q_{0i}(u)du B_i^1(t-u) = q_{0i}(t) © B_i^1(t)$$

After taking L. T. of the relations (82-85) and put in matrix form as follows:

$$\begin{vmatrix} B_0^{1*} \\ B_1^{1*} \\ B_2^{1*} \\ B_3^{1*} \end{vmatrix} = \begin{vmatrix} 1 & -q_{01}^* & -q_{02}^* & -q_{03}^* \\ -q_{10}^* & 1-q_{11}^{(4)*} & -q_{12}^{(5)*} & 0 \\ -q_{20}^* & -q_{21}^{(7)*} & 1-q_{22}^{(6)*} & 0 \\ -q_{30}^* & -q_{31}^{(8)*} & -q_{32}^{(9)*} & 1 \end{vmatrix}^{-1} \begin{vmatrix} 0 \\ Z_1^* + q_{14}^* Z_4^* + q_{15}^* Z_5^* \\ 0 \\ 0 \end{vmatrix} \quad (86)$$

Solve the above matrix for $B_i^{1*}(s)$, we get

$$B_0^{1*}(s) = \frac{N_3(s)}{D_2(s)} \quad (87)$$

Where,

$$N_3(s) = (Z_1^* + q_{14}^* Z_4^* + q_{15}^* Z_5^*)[q_{01}^*\left(1-q_{22}^{(6)*}\right) + q_{02}^* q_{21}^{(7)*} + -q_{03}^*\{q_{21}^{(7)*} q_{32}^{(9)*} + q_{31}^{(8)*}\left(1-q_{22}^{(6)*}\right)\}] \quad (88)$$

And $D_2(s)$ is same as in earlier in availability.

Now to obtain the steady state probability that the repairman is busy in repair of failed unit we proceed as follows:

$Z_i^*(0) = \int_0^\infty Z_i(t)dt = \Psi_i$ and using result $q_{ij}^*(0) = p_{ij}$

Therefore, in long run fraction of time for which system is under repair of Hardware failure is given by

$$B_0^1 = \frac{N_3(0)}{D_2(0)} = \frac{N_3}{D_2} \tag{89}$$

Where,

$$N_3 = N_3(0) = (\Psi_1 + p_{14}\Psi_4 + p_{15}\Psi_5)[p_{01}\left(1 - p_{22}^{(6)}\right) + p_{02}p_{21}^{(7)} + p_{03}\{p_{21}^{(7)}p_{32}^{(9)} + p_{31}^{(8)}\left(1 - p_{22}^{(6)}\right)] \tag{90}$$

And D_2 is same as in availability.

Now, expected busy period of the repairman in (0, t) is

$$\mu_b^1(t) = \int_0^t B_0^1 du \tag{91}$$

So that,

$$\mu_b^{1*}(s) = \frac{B_0^{1*}(s)}{s} \tag{92}$$

(b) Busy period due to Human error failure of repair: Let $B_i^2(t)$ be the probability that the repairman is busy in the repair of failed unit which is fail due to human error. Using elementary probabilistic arguments yield the following recursive relations.

$$B_0^2(t) = q_{01}(t) © B_1^2(t) + q_{02}(t) © B_2^2(t) + q_{03}(t) © B_3^2(t)$$
$$B_1^2(t) = q_{10}(t) © B_0^2(t) + q_{11}^{(4)}(t) © B_1^2(t) + q_{12}^{(5)}(t) © B_2^2(t)$$
$$B_2^2(t) = Z_2(t) + q_{26}(t) © Z_6(t) + q_{27}(t) © Z_7(t) + q_{20}(t) © B_0^2(t) + q_{21}^{(7)}(t) © B_1^2(t) + q_{22}^{(6)}(t) © B_2^2(t)$$
$$B_3^2(t) = q_{30}(t) © B_0^2(t) + q_{31}^{(8)}(t) © B_1^2(t) + q_{32}^{(9)}(t) © B_2^2(t) \tag{93-96}$$

After taking L. T. of the relations (93-96) and put in matrix form as follows:

$$\begin{vmatrix} B_0^{2*} \\ B_1^{2*} \\ B_2^{2*} \\ B_3^{2*} \end{vmatrix} = \begin{vmatrix} 1 & -q_{01}^* & -q_{02}^* & -q_{03}^* \\ -q_{10}^* & 1 - q_{11}^{(4)*} & -q_{12}^{(5)*} & 0 \\ -q_{20}^* & -q_{21}^{(7)*} & 1 - q_{22}^{(6)*} & 0 \\ -q_{30}^* & -q_{31}^{(8)*} & -q_{32}^{(9)*} & 1 \end{vmatrix}^{-1} \begin{vmatrix} 0 \\ 0 \\ Z_2^* + q_{26}^* Z_6^* + q_{27}^* Z_7^* \\ 0 \end{vmatrix} \tag{97}$$

Solve the above matrix for $B_i^{2*}(s)$, we get

$$B_0^{2*}(s) = \frac{N_4(s)}{D_2(s)} \tag{98}$$

Where,

$$N_4(s) = (Z_2^* + q_{26}^* Z_6^* + q_{27}^* Z_7^*)[q_{01}^* q_{12}^{(5)*} + q_{02}^*\left(1 - q_{11}^{(4)*}\right) + q_{03}^*\{\left(1 - q_{11}^{(4)*}\right)q_{32}^{(9)*} + q_{31}^{(8)*}q_{12}^{(5)*}\}] \tag{99}$$

And $D_2(s)$ is same as in earlier in availability.

Now to obtain the steady state probability that the repairman is busy in repair of failed unit we proceed as follows:

$Z_i^*(0) = \int_0^\infty Z_i(t)dt = \Psi_i$ and using result $q_{ij}^*(0) = p_{ij}$

Therefore, in long run fraction of time for which system is under repair of Human error failure is given by

$$B_0^2 = \frac{N_4(0)}{D_2(0)} = \frac{N_4}{D_2} \tag{100}$$

Where,

$$N_4 = N_4(0) = (\Psi_2 + p_{26}\Psi_6 + p_{27}\Psi_7)[p_{01}p_{12}^{(5)} + p_{02}\left(1 - p_{11}^{(4)}\right) + p_{03}\{\left(1 - p_{11}^{(4)}\right)p_{32}^{(9)} + p_{31}^{(8)}p_{12}^{(5)}\}] \tag{101}$$

And D_2 is same as in availability.

Now, expected busy period of the repairman in (0, t) is

$$\mu_b^2(t) = \int_0^t B_0^2 du \qquad (102)$$

So that,

$$\mu_b^{2*}(s) = \frac{B_0^{2*}(s)}{s} \qquad (103)$$

(c) Busy period due to Warm standby failure of repair: Let $B_i^s(t)$ be the probability that the repairman is busy in the repair of failed unit which is fail due to warm standby. Using elementary probabilistic arguments as in previous, we get

$$B_0^s = \frac{N_s}{D_2} \qquad (104)$$

Where,

$$N_s = (\Psi_3 + p_{38}\Psi_8 + p_{39}\Psi_9)[p_{03}\{(1 - p_{11}^{(4)})(1 - p_{22}^{(6)}) - p_{21}^{(7)} p_{12}^{(5)}\}] \qquad (105)$$

And D_2 is same as in availability.

Now, expected busy period of the repairman in (0, t) is

$$\mu_b^s(t) = \int_0^t B_0^s du \qquad (106)$$

So that,

$$\mu_b^{s*}(s) = \frac{B_0^{s*}(s)}{s} \qquad (107)$$

1.9 PROFIT ANALYSIS

The expected net gain incurred in (0, t) is defined as:

$P_0(t) =$ Expected total revenue earned by the system during (0, t)

- Expected total cost of repair during time interval (o, t)
- Expected total cost of preparation during time interval (o, t)

$$= K_0\mu_{up}(t) - K_1\mu_b^1(t) - K_2\mu_b^2(t) - K_3\mu_b^s(t) \qquad (108)$$

Where,

$K_0 =$ revenue per-unit up time.

$K_1 =$ Cost per-unit time when the repairman is busy in repair of failed unit due to hardware failure.

$K_2 =$ Cost per-unit time when the repairman is busy in repair of failed unit due to human error.

$K_3 =$ Cost per-unit time when the repairman is busy in repair of failed unit due to warm standby.

The expected profit per unit of time in steady state is given by

$$P_0 = \lim_{t\to\infty} \frac{P(t)}{t} = \lim_{s\to 0} s^2 p^*(s)$$

$$= K_0 \lim_{s\to 0} s^2\mu_{up}^*(s) - K_1 \lim_{s\to 0} s^2\mu_b^{1*}(s) - K_2 \lim_{s\to 0} s^2\mu_b^{2*}(s) - K_3 \lim_{s\to 0} s^2\mu_b^{s*}(s)$$

$$= K_0 \lim_{s\to 0} sA_0^*(s) - K_1 \lim_{s\to 0} sB_0^{1*}(s) - K_2 \lim_{s\to 0} s^2\mu_b^{2*}(s) - K_3 \lim_{s\to 0} s^2\mu_b^{s*}(s)$$

$$= K_0A_0 - K_1B_0^1 - K_2B_0^2 - K_3B_0^s \qquad (109)$$

Where A_0, B_0^1, B_0^2 and B_0^s are defined in (77), (89), (100) and (104) respectively.

CONCLUSION:

The reliability of a system without assuming human error failure may not depict a real picture of the actual reliability/availability modeling. Therefore the real time system reliability modeling must include the occurrence of common cause failures, hardware error and human error. The transient availability and other performance indices obtained may be helpful to improve the system availability in particular when occurrence of common cause failure and human errors are involved.

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